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APPLICATION OF DIFFERENTIAL EQUATIONS TO PHYSICAL EXAMPLES IN THE STUDY OF DAMPED VIBRATIONS.

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Annotation

Taking into account the differential equation of motion, since the tension force of the spring is balanced by the force of gravity in the state of inertia, the differential equation can be simplified and this equation represents the free oscillation of the load. represents It is called the equation of a harmonic oscillator and we clarify the physical meaning of e chim .

Key word

resistance force, differential equation of motion, harmonic oscillation, free oscillation, second-order linear homogeneous differential equation , Newton's second law , Hooke's law .

damping vibrations through differential equations, we have to work the following physical example . A load of mass Pb is suspended from a vertical spring of length b at rest . lThe load is pulled down a little and then released . Find the law of motion of the load , taking into account air resistance , which is proportional to the speed of movement .

Solving. Here, the force of air resistance is added to the series of forces acting on the load $R = -\mu\vartheta$(the negative sign means that **R**the force **ϑ**is in the opposite direction to the speed). The projection of the differential equation of motion Oxonto the axis will look like this:

$$m\frac{d^2x}{dt^2} = -cx - \mu\frac{dx}{dt}$$

or $c/m = k^2, \mu/m = 2n$say

$$\frac{d^2x}{dt^2} + 2n\frac{dx}{dt} + k^2x = 0\#(1)$$

This equation is also a second-order linear homogeneous equation with constant coefficients. Its $r^2 + 2nr + k^2 = 0$characteristic equation

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$$r_{1,2} = -n \pm \sqrt{n^2 - k^2}\#(2)$$

has roots.

The character of the movement is fully determined by these roots. Three things can happen. Let's look at the initial $n^2 - k^2 < 0$situation. This inequality is valid when the resistance of the medium is not very large. If $k^2 - n^2 = k_1^2$we say that (2) has the form of roots , the general solution is written in this form:$r_{1,2} = -n \pm ik_1$

$$x = e^{-nt}(C_1 \cos k_1 t + C_2 \sin k_1 t)$$

or rewritten as

$$x = Ae^{-nt}\sin(k_1 t + \alpha)\#(3)$$

If the initial conditions are given $t = 0$in $x_0 = x_0, \vartheta = \vartheta_0$, $A$and αcan be determined. For this

$$\vartheta = \frac{dx}{dt} = Ak_1 e^{-nt}\cos(k_1 t + \alpha) - Ane^{-nt}\sin(k_1 t + \alpha)$$

, $\vartheta$and $t = 0$substituting for and , we get this system of equations :x

$$x_0 = Asin\alpha,$$

$$\vartheta_0 = Ak_1 cosa - Ansin\alpha.$$

Divide both sides of the second equation by the corresponding sides of the first equation $\vartheta_0/x_0 = k_1 \operatorname{ctg}\alpha - n$to get:

from here

$\operatorname{ctg}\alpha = \frac{\vartheta_0 + nx_0}{k_1 x_0}$ or $\operatorname{tg}\alpha = \frac{k_1 x_0}{\vartheta_0 + nx_0}$, $\alpha = \operatorname{arctg}\frac{k_1 x_0}{\vartheta_0 + nx_0}$. It is known that

$$\sin\alpha = \frac{\operatorname{tg}\alpha}{\sqrt{1 + \operatorname{tg}^2\alpha}} = \frac{k_1 x_0/(\vartheta_0 + nx_0)}{\sqrt{1 + k_1^2 x_0^2/(\vartheta_0 + nx_0)^2}} = \frac{k_1 x_0}{\sqrt{k_1^2 x_0^2 + (\vartheta_0 + nx_0)^2}}$$

Therefore

$$A = \frac{x_0}{\sin\alpha} = \frac{\sqrt{k_1^2 x_0^2 + (\vartheta_0 + nx_0)^2}}{k_1}$$

(3) shows that we have a damped oscillation. Indeed , the oscillation amplitude $Ae^{-nt}$is a time-dependent and monotonically decreasing function, while $t \to \infty$at $Ae^{-nt} \to 0$.

The period of decaying oscillations is determined by this formula:

$$T = \frac{2\pi}{k_1} = \frac{2\pi}{\sqrt{k^2 - n^2}}$$

The difference of the moments of time when the load received the maximum deviation from the beginning of the coordinates (equilibrium state) $T/2$is an arithmetic progression equal to half a period. The amplitudes of the decaying oscillations $e^{-n\pi/k_1}$form a decreasing geometric progression equal to the denominator or . $e^{-nT/2}$This amount is called the fade decrement and is usually

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denoted $D$by a letter . The natural logarithm $\ln D = -nT/2$of the decrement is called the logarithmic decrement of the decay.

In this case, the frequency of oscillations $k_1 = \sqrt{k^2 - n^2}$is small ( ) compared to the previous case $k_1 < k$, but similar to that on the ground, it does not depend on the initial position of the load.

If numerical data are given: period of oscillation $T = 2\text{sec}$, decay decrement of oscillation $D = 1/2$as well as initial conditions $x_0 = 0$and $\vartheta_0 = 1$ m/s, then the law of motion of the load is found according to the formula (5) in this form:

$$x = Ae^{-nt}\sin(k_1 t + \alpha)$$

here $k_1$and $n$from the following relations:

$$T = 2\pi/k_1 = 2, \text{ бу ердан } k_1 = \pi$$

$$D = e^{-nT/2} = e^{-n} = 1/2, \text{ бу ердан } n = \ln 2$$

$t = 0$at $x_0 = 0$and $\vartheta_0 = 1$ m/sec Aallow us to determine the initial conditions and . We have the following:

$$\alpha = \text{arctg}\frac{k_1 x_0}{\vartheta_0 + n x_0} = 0, \ A = \frac{\vartheta_0}{k_1} = \frac{1}{\pi}$$

and finally:

$$x = \frac{\sin \pi t}{\pi 2^t}$$

If the resistance of the environment is large and $n^2 - k^2 > 0$that $n^2 - k^2 = h^2$, (2) roots$r_{1,2} = -n \pm h = -(n \mp h)$ we create in appearance. $h < n$since both roots are negative. In this case, the general solution of Eq

$$x = C_1 e^{-(n+h)t} + C_2 e^{-(n-h)t} \#(4)$$

will appear. From here it can be seen that the movement is non-periodic and non-oscillating in nature. $n^2 - k^2 = 0$the action will have a similar character, in which case the general solution is

$$x = e^{-nt}(C_1 + C_2 t)\#(5)$$

will have an appearance.

that $x \to 0$in the next two cases $t \to \infty$.

If $x(0) = x_0$and $x'(0) = \vartheta_0$are given initial conditions, $n^2 - k^2 > 0$then we have $x_0 = C_1 + C_2$and $\vartheta_0 = -(n+h)C_1 - (n-h)C_2$. Solving this system with respect to $C_1$and ,C_2

$$C_1 = \frac{x_0(h-n) - \vartheta_0}{2h}, \ C_2 = \frac{x_0(h+n) + \vartheta_0}{2h}$$

we find , so

$$x = e^{-nt}\left[\frac{x_0(h-n)-\vartheta_0}{2h}e^{-ht} + \frac{x_0(h+n)+\vartheta_0}{2h}e^{ht}\right] =$$

$$= e^{-nt}\left[\frac{x_0 n+\vartheta_0}{h}\frac{e^{ht}-e^{-ht}}{2} + x_0\frac{e^{ht}+e^{-ht}}{2}\right] =$$

$$= e^{-nt}\left(x_0 \operatorname{ch} ht + \frac{x_0 n+\vartheta_0}{h}\operatorname{sh} ht\right).$$

$n^2 - k^2 = 0$ бўлган ҳолда $C_1 = x_0, C_2 = x_0 n + \vartheta_0$ га эгамиз, ва бинобарин,

$$x = e^{-nt}[x_0 + (x_0 n + \vartheta_0)t]$$

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