

APPLICATION OF DIFFERENTIAL EQUATIONS TO PHYSICAL EXAMPLES IN THE STUDY OF HARMONIC VIBRATIONS.

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Annotation

Taking into account the differential equation of motion, since the tension force of the spring is balanced by the force of gravity in the state of inertia, the differential equation is simplified, and this equation is the free oscillation of the load . represents It is called the equation of a harmonic oscillator. This is a second-order linear homogeneous differential equation with constant coefficients. Its characteristic equation $r^2 + k^2 = 0$ will have abstract $r = \pm ik$ roots and clarify the physical meaning of e^{chim} .

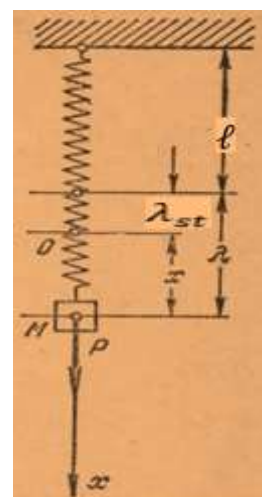
Keywords

differential equation of motion, harmonic oscillation, free oscillation, second order linear homogeneous differential equation , Newton's second law , Hooke's law .

Harmonic vibrations through differential equations, we have to work the following physical example . A load of mass P is suspended from a vertical spring of length b at rest . The load is pulled down a little and then released . Find the law of motion of the load, ignoring the mass of the spring and air resistance.

In solving this problem , we move the axis Ax down vertically y through the point where the load hangs . We take the coordinate head O at the point where the load is in equilibrium, that is, at the point where the weight of the load is balanced by the reaction force of the spring (Fig. 1).

λ - elongation of the spring at the same moment, λ_{st} and static elongation, that is, the distance from the end of the unstretched spring to the equilibrium position. Then $\lambda = \lambda_{st} + x$ or $\lambda - \lambda_{st} = x$.



motion from Newton's second law $F = ma$, where $m = P/g$ the mass of the load, a is the acceleration of the motion, and F is the equal force acting on the load. In this case, the equivalent force is the sum of the tension force of the spring and the force of gravity.

According to Hooke's law, the tension force of a spring is proportional to its elongation, i.e. equal to $-c\lambda$ called uniqueness.

Therefore, the differential equation of motion takes the following form:

$$m \frac{d^2x}{dt^2} = -c\lambda + P$$

Since $P = c\lambda_{st}$ the tension force of the spring in equilibrium is balanced by the force of gravity will be. By putting k^2 the expression of $\lambda - \lambda_{st}$ the differential equation and P denoting $m \frac{d^2x}{dt^2} = -cx$ by x

$$\frac{d^2x}{dt^2} + k^2x = 0 \quad (1)$$

we will show.

This equation represents the so-called *free swing* λ of the load. It is called the equation of a harmonic oscillator. This is a second-order linear homogeneous differential equation with constant coefficients. Its characteristic equation has $r^2 + k^2 = 0$

abstract roots $r = \pm ik$, the corresponding general solution is:

$$x = C_1 \cos kt + C_2 \sin kt$$

It is more convenient to reformulate the solution by introducing new arbitrary variables to $\sqrt{C_1^2 + C_2^2}$ determine the physical meaning of the solution. Multiplying and dividing by $\sqrt{C_1^2 + C_2^2}$, we get the following:

$$x = \sqrt{C_1^2 + C_2^2} \left(\frac{C_1}{\sqrt{C_1^2 + C_2^2}} \cos kt + \frac{C_2}{\sqrt{C_1^2 + C_2^2}} \sin kt \right)$$

If

$$\sqrt{C_1^2 + C_2^2} = A, \quad C_1/\sqrt{C_1^2 + C_2^2} = \sin \alpha, \quad C_2/\sqrt{C_1^2 + C_2^2} = \cos \alpha$$

solution

$$x = A \sin(kt + \alpha) \quad (2)$$

appears.

Thus, the load oscillates harmonically around the equilibrium position.

The quantity A is called the amplitude of the oscillation, $kt + \alpha$ and the argument is called the phase of the oscillation. The value of the phase $t = 0$, that is, the initial phase of the quantity oscillation, is called α . $k = \sqrt{c/m}$ magnitude is the frequency of oscillation. The period and frequency k of oscillation $T = 2\pi/k =$

$2\pi\sqrt{m/c}$ depends only on the stiffness of the spring and the mass of the system.

$c = P/\lambda_{st} = mg/\lambda_{st}$ since, for the period

$$T = 2\pi\sqrt{\lambda_{st}/g}$$

the formula can also be derived.

the speed of movement of the load t by differentiating the solution:

$$\vartheta = \frac{dx}{dt} = Ak\cos(kt + \alpha)$$

Initial conditions must be given to determine the amplitude and initial phase.

Let be the position $x = x_0$ and velocity of the load at $\vartheta = \vartheta_0$ the initial moment $t = 0$.

In that case

$$x_0 = A\sin \alpha, \vartheta_0 = Ak\cos \alpha, \text{ from here}$$

$$A = \sqrt{x_0^2 + \frac{\vartheta_0^2}{k^2}}, \alpha = \arctg \frac{kx_0}{\vartheta_0}.$$

As can be seen from the amplitude and initial phase formulas, they depend on the initial state of the system, as opposed to the frequency and period of the individual oscillations. In the absence of initial velocity ($\vartheta_0 = 0$), the amplitude is $A = x_0$, and the initial phase is $\alpha = \pi/2$ and so,

$$x = x_0 \sin\left(kt + \frac{\pi}{2}\right) \text{ ёки } x = x_0 \cos kt$$

If numerical data is given, e.g. $P = 2H, l = 40sm, \lambda_{st} = 4 sm$, at the same time $x = 2 sm$ being pulled by the load, without initial speed ($\vartheta_0 = 0$) If q is carved, then the law of motion is according to the formula (2).

$$x = A\sin(kt + \alpha)$$

is defined in the form, where $k = \sqrt{cg/P}$ the frequency $P = c\lambda_{st}$ is determined from the relation, from where $c = 1/2$, therefore, $k = \sqrt{g/2}, A = \sqrt{x_0^2 + \vartheta_0^2/k^2} = 2$ and $\alpha = \arctg(kx/\vartheta_0) = \pi/2$. so that

$$x = 2\cos \frac{t}{2}\sqrt{g}.$$

The period of vibration of the load $T = 2\pi/k = 4\pi/\sqrt{g} \approx 0,4sek$. The greatest elongation is $\lambda_{max} = \lambda_{st} + A = 6sm$ the greatest tension force of the spring $F_{max} = c\lambda_{max} = 3N$.

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