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TOʻLDIRUVCHI FAZONING OʻLCHAMI MAKSIMAL NILRADIKALI FILIFORM BOʻLGAN YECHILUVCHAN LI ALGEBRASINING DIFFERENSIALLASHLARI

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Annotatsiya

Bu maqolada nilradikali filiform va toʻldiruvchi fazosining oʻlchami maksimal boʻlgan yechilivchan Li algebralaridagi har qanday lokal differensiallash differensiallash boʻlishi koʻrsatilgan.

Kirish

Xarakteristikasi nolga teng algebraik yopiq maydon ustida chekli oʻlchamli Li algebralarida lokal va 2-lokal differensiallashlar va avtomorfizmga oid yurtimizda bir qator ilmiy izlanishlar olib borilib, dastlabki natijalar Sh.A.Ayupov, K.K.Kudaybergenov, I.S.Raximovlarga tegishlidir [7]. Ular tomonidan yarim sodda Li algebrasidagi har bir 2-lokal differensiallash oddiy maʼnoda differensiallash ekanligi va oʻlchami ikkidan katta ixtiyoriy nilpotent Li algebrasida differensiallash boʻlmagan 2-lokal differensiallash mavjudligini isbotlangan. Keyinchalik Sh.A.Ayupov va K.K.Kudaybergenovlar yarim sodda Li algebralaridagi har bir lokal differensiallash ham oddiy maʼnoda differensiallash ekanligini isbotladilar va differensiallash boʻlmagan lokal differensiallashi mavjud boʻlgan chekli oʻlchamli nilpotent Li algebralariga misollar keltirdilar [2]. Sodda Li algebralar uchun 2-lokal avtomorfizmlar Z.Chen va D. Vanglar tomonidan oʻrganila boshlanib, ular tomonidan A_l, D_l yoki $E_k, (k = 6,7,8)$ algebralar uchun har qanday 2-lokal avtomorfizm avtomorfizm ekanligi isbotlangan [8]. Keyinchalik, Sh.A.Ayupov va K.K.Kudaybergenovlar tomonidan ushbu natija umumlashtirilib, xarakteristikasi nol boʻlgan algebraik yopiq maydon ustuda berilgan ixtiyoriy chekli oʻlchamli yarim sodda Li algebrasining har qanday 2-lokal avtomorfizmi avtomorfizm ekanligini isbotlandi[8]. Bundan tashqari, ixtiyoriy chekli oʻlchamli nilpotent Li algebrasida avtomorfizm boʻlmagan 2-lokal avtomorfizm mavjudligini koʻrsatildi. Sh.A.Ayupov va K.K.Kudaybergenovlar baʼzi chekli oʻlchamli sodda Li va Leibniz algebralarining lokal avtomorfizmlarini oʻrganganlar[4].

Ta'rif-1. Agar $d: L \to L$ chiziqli akslantirish uchun quyidagi tenglik bajarilsa:

$$d([x,y]) = [d(x),y] + [x,d(y)] \qquad \forall x,y \in L$$

U holda ushbu akslantirish berilgan L algebrasining differensiallashi deyiladi.

Ta'rif-2. Bizga $\Delta: L \to L$ chiziqli akslantirish berilgan bo'lsin. Agar L algebraning $\forall x \in L$ elementi uchun shunday D_x differensiallash topilib $\Delta(x) = D_x(x)$ tenglik bajarilsa, u holda Δ lokal differensiallash deyiladi

Yechiluvchan va filiform Li algebrasi

Ixtiyoriy L Li algebrasi uchun quyida ketma-ketliklari aniqlaymiz:

$$L^{[1]} = L,\ L^{[k+1]} = \left[L^{[k]}, L^{[k]}\right],\ k \geq 1$$

$$L^{1} = L,\ L^{k+1} = \left[L^{k}, L^{1}\right],\ k \geq 1.$$

Agar L Li algebrasi uchun shunday $m \in N$ son mavjud bo'lib, $L^{[m]} = 0$ bo'lsa, u holda L yechimli Li algebrasi deyiladi. Ana shunday m larning eng kichigini L yechimli Li algebrasining indeksi deyiladi.

Agar L Li algebrasi uchun shunday $s \in N$ son mavjud bo'lib, $L^{s} = 0$ bo'lsa u holda L nilpotent Li algebrasi deyiladi. Ana shunday xususiyatga ega bo'lgan minimal s soni nilpotentlik indeksi yoki L Li algebrasining nilindekisi deyiladi.

L Li algebrasining maksimal nilpotent idealiga uning nilradikali deyiladi.

Nilradikali filiform bo'lgan yechiluvchan Li algebrasining differensiallashlari.

Har qanday tabiiy ravishda baholangan filiform Li algebra quyidagi izomorf bo'lmagan algebralardan biriga izomorfdir:

$$n_{n,1}: \{[e_i, e_1] = -[e_1, e_i] = e_{i+1}, \quad 2 \leq i \leq n-1.$$

$$Q_{2n}: \begin{cases} [e_i, e_1] = -[e_1, e_i] = e_{i+1}, & 2 \leq i \leq 2n-2, \\ [e_i, e_{2n+1-i}] = -[e_{2n+1-i}, e_i] = (-1)^i e_{2n}, & 2 \leq i \leq n. \end{cases}$$

Nilradikal tabiiy ravishda baholangan filiform Li algebra bo'lgan barcha hal qilinadigan Li algebralari $n_{n,1}$ [53] da tasniflanadi. Nilradikal tabiiy ravishda baholangan filiform Lie algebra Q_{2n} bo'lgan yanada hal qilinadigan Li algebralari [2] da tasniflanadi. Nilradikal uchun izomorfik bo'lgan hal qilinadigan Li algebrasining o'lchami isbotlangan n-o'lchovli tabiy ravishda baholangan filiform Li algebra dan katta emas $n+2$. Bu yerda biz bunday hal qilinadigan Li algebralari ro'yxatini beramiz. $s_{n,2}$ orqali $n_{n,1}$ niradikali yechiuvchan Li algebralarini belgilaymiz. Xuddi shunday Q_{2n} nilradikalli Li algebralari uchun $\tau_{2n,2}$ belgilash olamiz.

$$
s_{n,2}:\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le n-1,\\ [e_1,x]=e_1, & \\ [e_i,x]=(i-2)e_i, & 3\le i\le n,\\ [e_i,y]=e_i, & 2\le i\le n.\end{cases}
\qquad
\tau_{2n,2}:\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le 2n-2,\\ [e_i,e_{2n+1-i}]=(-1)^i e_{2n}, & 2\le i\le n,\\ [e_i,x]=ie_i\ , & 1\le i\le 2n-1,\\ [e_{2n},x]=(2n+1)e_{2n}, & \\ [e_i,y]=e_i, & 1\le i\le 2n-1,\\ [e_{2n},y]=-[y,e_{2n}]=2e_{2n}. & \end{cases}
$$

Quyidagi teorema ushbu bo‘limning asosiy natijasidir.

Teorema. $s_{n,2}$ va $\tau_{2n,2}$ algebralarning differensiallari quyidagi shaklga ega:

$$
Der(s_{2n,2}):\begin{cases}d(e_1)=a_1e_1+\sum_{i=3}^{n}a_ie_i,\\ d(e_2)=b_2e_2-c_1e_3,\\ d(e_i)=(b_2+(i-2)a_1)e_i-c_1e_{i+1}, \quad 3\le i\le n\text{-}1,\\ d(e_n)=(b_2+(n-2)a_1)e_n,\\ d(x)=c_1e_1+\sum_{i=3}^{n-1}(i-2)a_{i+1}e_i+c_ne_n,\\ d(y)=\sum_{i=2}^{n-1}a_{i+1}e_i+\dfrac{c_n}{(n-2)}e_n,\end{cases}
$$

$$
Der(\tau_{2n,2}):\begin{cases}d(e_1)=(a_{2n+1}+a_{2n+2})e_1,\\ d(e_i)=(ia_{2n+1}+a_{2n+2})e_i+a_1e_{i+1}+(-1)^i a_{2n+1-i}e_{2n}, & 2\le i\le n\\ d(e_i)=(ia_{2n+1}+a_{2n+2})e_i+a_1e_{i+1}, & n+1\le i\le 2n-2\\ d(e_{2n-1})=(a_{2n+1}+a_{2n+2})e_{2n-1},\\ d(e_{2n})=((2n+1)a_{2n+1}+2a_{2n+2})e_{2n},\\ d(y)=-2a_{2n}e_{2n}.\end{cases}
$$

Isbot. Biz $s_{n,2}$ algebra uchun isbotni kiritamiz , $\tau_{2n,2}$ algebra uchun isbot o‘xshash. d $s_{n,2}$ning differensiali bo‘lsin. Biz o‘rnatdik

$$
d(e_1)=\sum_{i=1}^{n}a_ie_i+a_{n+1}x+a_{n+2}y, \qquad d(e_2)=\sum_{i=1}^{n}b_ie_i+b_{n+1}x+b_{n+2}y,
$$

$$
d(x)=\sum_{i=1}^{n}c_ie_i+c_{n+1}x+c_{n+2}y, \qquad d(y)=\sum_{i=1}^{n}\lambda_ie_i+\lambda_{n+1}x+\lambda_{n+2}y.
$$

Bundan tashqari bizda

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$$d(e_1)=d[e_1,x]=[d(e_1),x]+[e_1,d(x)]=\left[\sum_{i=1}^{n}a_ie_i+a_{n+1}x+a_{n+2}y,x\right]+\left[e_1,\sum_{i=1}^{n}c_ie_i+c_{n+1}x+c_{n+2}y\right]=$$
$$=a_1e_1+\sum_{i=2}^{n}a_i(i-2)e_i-\sum_{i=2}^{n-1}c_ie_{i+1}+c_{n+1}e_1=a_1e_1+a_3e_3+2a_4e_4+3a_5e_5+$$
$$+\ldots+(n-2)a_ne_n-c_2e_3-c_3e_4-c_4e_5-\ldots-c_{n-1}e_n+c_{n+1}e_1=$$
$$=(a_1+c_{n+1})e_1+(a_3-c_2)e_3+(2a_4-c_3)e_4+\ldots+((n-2)a_n-c_{n-1})e_n,$$

Boshqa tomondan,

$$d(e_1)=a_1e_1+a_2e_2+\ldots+a_ne_n+a_{n+1}x+a_{n+2}y.$$

Shuning uchun,

$$\begin{cases} a_2=c_2=a_{n+1}=a_{n+2}=c_{n+1}=0 \\ c_3=a_4 \\ c_4=2a_5 \\ \ldots\ \ldots\ \ldots \\ c_{n-1}=(n-3)a_n \end{cases}$$

Quyidagi shartdan

$$0=d[e_1,y]=[d(e_1),y]+[e_1,d(y)]=\left[a_1e_1+\sum_{i=3}^{n}a_ie_i,y\right]+\left[e_1,\sum_{i=1}^{n}\lambda_ie_i+\lambda_{n+1}x+\lambda_{n+2}y\right]=$$
$$=\sum_{i=3}^{n}a_ie_i-\sum_{i=2}^{n-1}\lambda_ie_{i+1}+\lambda_{n+1}e_1=a_3e_3+a_4e_4+\ldots+a_ne_n-\lambda_{n+1}e_1-\lambda_2e_3-$$
$$-\lambda_4e_5-\ldots-\lambda_{n-1}e=\lambda_{n+1}e_1+(a_3-\lambda_2)e_3+(a_4-\lambda_3)e_4+\ldots+(a_n-\lambda_{n-1})e_n,$$

Ushbu natijani olamiz

$$\begin{cases} \lambda_{n+1}=0 \\ \lambda_2=a_3 \\ \lambda_3=a_4 \\ \ldots\ \ldots\ \ldots \\ \lambda_{n-1}=a_n \end{cases}.$$

Quyidagini ko‘rib chiqamiz

$$0=d[x,y]=[d(x),y]+[x,d(y)]=\left[c_1e_1+\sum_{i=3}^{n-1}(i-2)a_{i+1}e_i+c_ne_n+c_{n+2}y,y\right]+$$
$$+\left[x,\lambda_1e_1+\sum_{i=2}^{n-1}a_{i+1}e_i+\lambda_ne_n+\lambda_{n+2}y\right]=\sum_{i=3}^{n-1}(i-2)a_{i+1}e_i+c_ne_n-\lambda_1e_1-$$
$$-\sum_{i=2}^{n-1}(i-2)a_{i+1}e_i-(n-2)\lambda_ne_n=(c_n-(n-2)\lambda_n)e_n-\lambda_1e_1.$$

Bundan

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$$\begin{cases} \lambda_n = \dfrac{c_n}{(n-2)}. \\ \lambda_1 = 0 \end{cases}$$

ega bo‘lamiz.

Endi quyidagini ko‘rib chiqamiz,

$$d(e_2) = d[e_2, y] = \left[\sum_{i=1}^{n} b_i e_i + b_{n+1}x + b_{n+2}y, y\right] + \left[e_2, \sum_{i=2}^{n-1} a_{i+1}e_i + \frac{c_n}{(n-2)}e_n + \lambda_{n+2}y\right] =$$

$$= \sum_{i=2}^{m} b_i e_i + \lambda_{n+2}e_2 = b_2e_2 + b_3e_3 + \text{K} + b_ne_n + \lambda_{n+2}e_2.$$

boshqa tomondan,

$$d(e_2) = b_1e_1 + b_2e_2 + \text{K} + b_ne_n + b_{n+1}x + b_{n+2}y.$$

Shuning uchun,

$$b_1 = b_{n+1} = b_{\partial n+2} = \lambda_{n+2} = 0.$$

Ushbu tenglikdan

$$0 = d[e_2, x] = \left[\sum_{i=2}^{n} b_i e_i, x\right] + \left[e_2, c_1e_1 + \sum_{i=3}^{n-1}(i-2)a_{i+1}e_i + c_ne_n + c_{n+2}y\right] = \sum_{i=2}^{n}(i-2)b_ie_i + c_1e_3 + c_{n+2}e_2$$

,

Quyidagiga ega bo‘lamiz,

$$b_i = 0,\ i = \overline{4,n},\ c_{n+2} = 0,\ \ b_3 = -c_1.$$

Demak, $s_{n,2}$ algebraning har qanday differensiali qiymatga ega degan xulosaga kelamiz.

$$d(e_1) = a_1e_1 + \sum_{i=3}^{n} a_ie_i,$$

$$d(e_2) = b_2e_2 - c_1e_3,$$

$$d(e_i) = (b_2 + (i-2)a_1)e_i - c_1e_{i+1},\ \ 3 \le i \le n-1,$$

$$d(e_n) = (b_2 + (n-2)a_1)e_n,$$

$$d(x) = c_1e_1 + \sum_{i=3}^{n-1}(i-2)a_{i+1}e_i + c_ne_n,$$

$$d(y) = \sum_{i=2}^{n-1} a_{i+1}e_i + \frac{c_n}{(n-2)}e_n.$$

FOYDALANILGAN ADABIYOTLAR RO'YXATI.

1. Ayupov Sh. A., Kudaybergenov K. K. 2-local derivations on von Neumann algebras // Positivity. 19 (2015), p. 445-455.

2. Ayupov Sh. A., Kudaybergenov K. K. Local derivations on finite-dimensional Lie algebras // Linear Algebra and its Applications. 493 (2016), p. 381–398.

3. Ayupov Sh. A., Kudaybergenov K. K. 2-Local automorphisms on finite-dimensional Lie algebras // Linear Algebra and its Applications. 507 (2016), p. 121–131.

4. Ayupov Sh. A., Kudaybergenov K. K. Local Automorphisms on Finite-Dimensional Lie and Leibniz Algebras // Algebra, Complex Analysis and Pluripotential Theory, USUZCAMP 2017. Springer Proceedings in Mathematics and Statistics. 264 (2017), p. 31–44.

5. Ayupov Sh. A., Kudaybergenov K. K., Nurjanov B., Alauadinov A. Local and 2-Local derivations on noncommutative arens algebras // Mathematic Slovaca. 64 (2014), p. 423–432.

6. Ayupov Sh. A., Kudaybergenov K. K., Omirov A. B. Local and 2-local derivations and automorphisms on simple Leibniz algebras // Bulletin of the Malaysian Mathematical Sciences Society. 43 (3) (2020), p. 2199–2234.

7. Ayupov Sh. A., Kudaybergenov K. K., Rakhimov I. S. 2-Local derivations on finite-dimensional Lie algebras // Linear Algebra and its Applications. 474 (2015), p. 1–11.

8. Chen Z., Wang D. 2-Local automorphisms of finite-dimensional simple Lie algebras // Linear Algebra and its Applications. 486 (2015), p. 335–344.

9. Šnobl L., On the structure of maximal solvable extensions and of Levi extensions of nilpotent Lie algebras // J. Phys. A, Math. Theor. 43:17 (2010) Article ID 505202.

10. Kadison R. V. Local derivations // Journal of Algebra. 130 (1990), p. 494–509.

11. B. Verbeke, Almost inner derivations of Lie algebras, PhD dissertation, KU Leuven Faculty of Science, 2020.

12. Kudaybergenov K.K., Omirov B.A., Kurbanbaev T.K. Local derivations on solvable Lie algebras of maximal rank // Communications in Algebra https://doi.org/10.1080/00927872.2022.2045494.

13. I.Kaygorodov, K.K.Kudaybergenov, I.G’.Yuldashev. Local derivations of semisimple Leibniz algebras // CIMPA School "Nonassociative Algebras and Its Applications", Madagascar. Communications in Mathematics 30 (2022), DOI: https://doi.org/10.46298/cm.9274

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14. I.G'.Yuldashev. Local derivations on solvable Lie algebras with a filiform nilradical // "Qoraqalpog'istonda fan va ta'lim" 2021-yil 4-son (18)