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ARIFMETIK PROGRESSIYAGA DOIR UCHRAYDIGAN MASALALAR VA ULARNI YECHISH.

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Annotatsiya

Maqolada sonli ketma-ketlik haqidagi muhim tushinchalar keltirib o'tilgan. Ketma-ketliklarning ko'p uchraydigan ko'rinishi bo'lgan arifmetik progressiya, uning xossalari va formulalari keltirilgan. Shuningdek, keltirilgan tushunchalarning amaliy masalalarga qo'llanilishi keltirib o'tilgan.

PROBLEMS OF ARITHMETIC PROGRESSION AND THEIR SOLUTION.

Annotatsiya

В статье упоминаются важные понятия о числовых последовательностях. Представлены арифметическая прогрессия, являющаяся распространенной формой последовательностей, ее свойства и формулы. Также упоминается применение представленных концепций к практическим вопросам.

ЗАДАЧИ АРИФМЕТИЧЕСКОЙ ПРОГРЕССИИ И ИХ РЕШЕНИЕ.

Annotation

In the article, important concepts about numerical sequences are mentioned. Arithmetic progression, which is a common form of sequences, its properties and formulas are presented. The application of the presented concepts to practical issues is also mentioned.

Kalit so'zlar

Ketma-ketlik, arifmetik progressiya, hadlar, ayirma, umumiy hadi, hadlar yig'indisi, rekkerent qoida, xossa.

Ключевые слова

Последовательность, арифметическая прогрессия, члены, разность, общий член, сумма членов, повторяющееся правило, свойство.

Keywords

Sequence, arithmetic progression, terms, difference, common term, sum of terms,

recurring rule, property.

Har bir natural n soniga ma'lum bir qoidaga ko'ra qandaydir a_n haqiqiy son mos qo'yilgan bo'lsa, u holda sonli ketma-ketlik berilgan deyiladi va quyidagi ko'rinishda yoziladi:

$a_1, a_2, a_3, \dots, a_n$ yoki $\{a_n\}$. Bu yerda $a_1, a_2, a_3, \dots, a_n$ sonlar ketma-ketlikning hadlari deyiladi.

Masalan:

I. $2, 4, 6, \dots\ , 2n$ *sonlarni qarasak, har bir natural songa juft son mos keltirilgan, ya'ni*

$a_n = 2n,\ (n \in N)$.

II. $1, 4, 9, 16, \dots, n^2$ *sonlarni qarasak, har bir natural songa shu sonning kvadrati mos keltirilgan, ya'ni*

$a_n = n^2,\ (n \in N)$.

Sonli ketma-ketliklar quyidagi ko'rinishlarda berilishi mumkin:

a) Sonli ketma-ketlik umumiy hadi formulasi bilan beriladi va bu formula yordamida uning ixtiyoriy hadini topish mumkin.

Masalan:

I. $a_n = 2n - 1,\ (n \in N)$ *formula bilan berilgan ketma-ketlikda n ga qiymatlar berib uning hadlarini aniqlaymiz:*

$$a_1 = 1, a_2 = 3, a_3 = 5, \dots$$

II. $a_n = (-1)^n, (n \in N)$ *formula bilan berilgan ketma-ketlikda n ga qiymatlar berib uning hadlarini aniqlaymiz:*

$$a_1 = -1, a_2 = 1, a_3 = -1, a_4 = 1, \dots$$

Rekurent qoida asosida, ya'ni ketma-ketlikning dastlabki bir nechta hadalari berilgan bo'lib, keying hadlari ular orqali aniqlanadi.

Masalan:

I. $2, 5, 8, \dots$ *ketma-ketlikda dastlabki hadlari berilgan bo'lib, ularning har biri ikkinchi hadidan boshlab bitta oldingi hadiga 3 ni qo'shish orqali hosil qilingan.*

II. $1, 1, 2, 3, 5, 8, 13, \dots$ *ketma-ketlikda uchinchisidan boshlab uning oldingi ikkita hadini qo'shish orqali keyingi hadi hosil qilinmoqda.*

Ketma-ketlik uning hadlariga xos bo'lgan umumiy qoida asosida berilishi mumkin.

Masalan: $\sqrt{3}$ *sonining 0,1; 0,01; 0,001;… aniqlikdagi ildizlari ketma-ketligi quyidagi ko'rinishda bo'ladi.*

1,7; 1,73; 1,732; …

Hadlarining soni chekli bo'lgan ketma-ketlik chekli ketma-ketlik, hadlarining soni cheksiz bo'lgan ketma-ketlik cheksiz ketma-ketlik deyiladi.

Masalan:

I. *Toq raqamlar ketma-ketligi: 1, 3, 5, 7, 9 chekli ketma-ketlik.*

II. *3 ga qoldiqsiz bo'linadigan natural sonlar ketma-ketligi: 3, 6, 9, 12, 15, 18, … cheksiz ketma-ketlik.*

Agar $a_1, a_2, a_3, \dots, a_n$ sonli ketma-ketlikda uning ikkinchi hadidan boshlab har bir hadi oldingisiga aniq bir o'zgarmas sonni qo'shish orqali hosil qilingan bo'lsa, u holda bu ketma-ketlik arifmetik progressiya deyiladi.

Qo'shiluvchi o'zgarmas son arifmetik progressiyaning ayirmasi deyiladi va d harfi bilan belgilanadi.

$$d = a_n - a_{n-1}$$

Arifmetik progressiya $d > 0$ da o'suvchi va $d < 0$ da kamayuvchi bo'ladi.

Masalan: $1, 3, 5, 7, 9, \dots$ *ketma-ketlik arifmetik progressiyani tashkil etib, bu yerda ayirma*

$d = a_2 - a_1 = 3 - 1 = 2.$

7. Ta'rifga asosan quyidagilar o'rinli:

$$a_2 = a_1 + d$$
$$a_3 = a_2 + d = a_1 + d + d = a_1 + 2d$$
$$a_4 = a_3 + d = a_1 + d + d + d = a_1 + 3d$$
$$\vdots$$

$a_n = a_1 + \underbrace{d + d + \cdots + d}_{(n-1)ta} = a_1 + (n-1)d.$

Demak, arifmetik progressiyaning ixtiyoriy hadini topish formulasi

$$a_n = a_1 + (n-1)d$$

ko'rinishda bo'ladi.

Masalan:

I. *Arifmetik progressiyada* $a_4 = 15$ *ga va* $d = 2{,}5$ *teng bo'lsa,* a_{15} *ni toping.*

Yechish: *berilgan masalada* $a_{15} = a_1 + 14d$ *ga teng bo'ladi. Demak, so'ralgan hadni topish uchun* a_1 *ni topish kerak:*

$a_4 = a_1 + 3d$

$$a_1 + 3 \cdot 2{,}5 = 15$$
$$a_1 = 7{,}5$$

$a_{15} = 7{,}5 + 14 \cdot 2{,}5 = 42{,}5.$

II. *Arifmetik progressiyaning beshinchi hadi 24 ga, o'n to'qqizinchi hadi esa 66 ga teng. Uning sakkizinchi hadini toping.*

Yechish: masalada berilganlariga ko'ra

$\begin{cases} a_5 = 24 \\ a_{19} = 66 \end{cases}$

$$\begin{cases} a_1 + 4d = 24 \\ a_1 + 18d = 66 \end{cases}$$

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$d = 3$,

$$a_1 = 12$$

Topilganlarga asosan a_8 ni aniqlaymiz:

$$a_8 = a_1 + 7d = 12 + 7 \cdot 3 = 33$$

Arifmetik progressiyaning o'rta hadi quyidagi formula bilan aniqlanadi:

$a_n = \frac{a_{n-k}+a_{n+k}}{2}$.

Masalan: *Arifmetik progressiyada $a_7 = 12$ ga va $a_{17} = 42$ ga teng. Uning o'n ikkinchi hadini toping.*

Yechish: *berilganlarga asosan quyidagini yozish mumkin:*

$a_{12} = \frac{a_{12-5}+a_{12+5}}{2} = \frac{a_7+a_{17}}{2} = \frac{12+42}{2} = 27.$

Agar arifmetik progressiyaning a_n, a_k, a_l, a_s ixtiyoriy hadlari uchun $n + s = k + l$ bo'lsa, u holda $a_n + a_s = a_k + a_l$ tenglik bajariladi.

Masalan: *Arifmetik progressiyada*

$a_3 + a_{13} = 12$ *ga va* $a_1^2 + a_{15}^2 = 96$ *ga teng bo'lsa,* $a_1 \cdot a_{15}$ *ni toping.*

Yechish: *yuqoridagi xossaga asosan*

$$a_3 + a_{13} = a_1 + a_{15} = 12$$

tenglik o'rinli. Berilganlardan foydalanib quyidagini aniqlaymiz:

$$a_1^2 + a_{15}^2 = (a_1 + a_{15})^2 - 2a_1 \cdot a_{15}$$

$12^2 - 2a_1 \cdot a_{15} = 96$

$a_1 \cdot a_{15} = 24.$

Arifmetik progressiyaning dastlabki n ta hadi yig'indisi, ya'ni

$$a_1 + a_2 + a_3 + \cdots + a_n = S_n$$

quyidagi formulalardan biri yordamida aniqlanadi:

I. $S_n = \frac{a_1+a_n}{2} \cdot n$

II. $S_n = \frac{2a_1+(n-1)\cdot d}{2} \cdot n$

Masalan:

I. *Arifmetik progressiyada* $a_1 = 6$ *va*

$a_{17} = 54$ *bo'lsa uning dastlabki yigirma beshta hadi yig'indisini toping.*

Yechish: *berilgan masaladagi* $a_{17} = 54$ *dan ayirma d ni aniqlab olamiz:*

$a_1 + 16d = 54,\ \ 6 + 16d = 54\ , d = 3$

Topilganlarga asosan progressiyaning dastlabki yigirma beshta hadi yig'indisini topamiz:

$S_{25} = \frac{2\cdot 6+(25-1)\cdot 3}{2} \cdot 25 = 1050.$

II. *Arifmetik progressiyada* $a_3 = 11$ *va*

$a_{19} = 45$ *bo'lsa uning dastlabki yigirma bitta hadi yig'indisini toping.*

Yechish: *berilgan hadlarni quyidagi ko'rinishda yozamiz:*

$$\begin{cases} a_3 = a_1 + 2d \\ a_{19} = a_1 + 18d \end{cases}$$

$$\begin{cases} a_1 + 2d = 11 \\ a_1 + 18d = 45 \end{cases}.$$

Hosil bo'lgan tenglamalar sistemasidagi tenglamalarni hadlab qo'shamiz:

$$+\begin{cases} a_1 + 2d = 11 \\ a_1 + 18d = 45 \end{cases}$$

$2a_1 + 20d = 56.$

Endi arifmetik progressiyaning dastlabki yigirma bitta hadini yig'indisini aniqlaymiz:

$S_{21} = \frac{2a_1+20\cdot d}{2} \cdot n = \frac{56}{2} \cdot 21 = 588.$

Arifmetik progressiyaning dastlabki n ta hadi yig'indisi S_n qiyidagi xosslarga ega:

I. $S_n - S_{n-1} = a_n$

II. $S_{n+1} - S_n = a_{n+1}$

III. $S_n - S_{n-2} = a_n + a_{n-1}$

Masalan: *Arifmetik progressiyada* $S_{28} = 740$, $S_{29} = 790$ *va* $d = 1{,}5$ *ga teng bo'lsa, uning birinchi hadini toping.*

Yechish: *berilgan masalada*

$S_{29} - S_{28} = a_{29}$

$$a_{29} = 490 - 440 = 50$$

$a_{29} = a_1 + 28d$

$$a_1 + 28 \cdot 1{,}5 = 50$$

$a_1 = 8$.

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