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CALCULATION METHODS OF SECOND AND THIRD ORDER DETERMINANTS

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Zaparov Ziyadilla Abdumalikovich
Senior teacher of Andijan Institute of Agriculture and Agro-Technology
ziyodulla19662410@gmail.com

Annotation

This article deals with determinants in a math course, specifically second and third order determinants. There are several ways to calculate them.

Keywords

Square matrix, determinant, Sarrius method, minor, algebraic complement, determinant of order n.

Introduction

The following $a_{11},\ a_{12},\ a_{21},\ a_{22}$ composed of real numbers $A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ a square table is called a square matrix of order 2, here a_{ij}- its elements, $a_{11},\ a_{12}$ *va* $a_{21},\ a_{22}$ are its string elements, $a_{11},\ a_{21}$ *va* $a_{12},\ a_{22}$ **called column elements**. The first index of a_{ij} is the row number *i*, *j* represents the column number. For example, a_{21} located in row 2 and column 1. We call the following number the determinant of this matrix:

$$\det A=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}=a_{11}a_{22}-a_{12}a_{21} \qquad (1)$$

Likewise,

$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ if we call a square table a square matrix of the 3rd order, we say the following number as its determinant:

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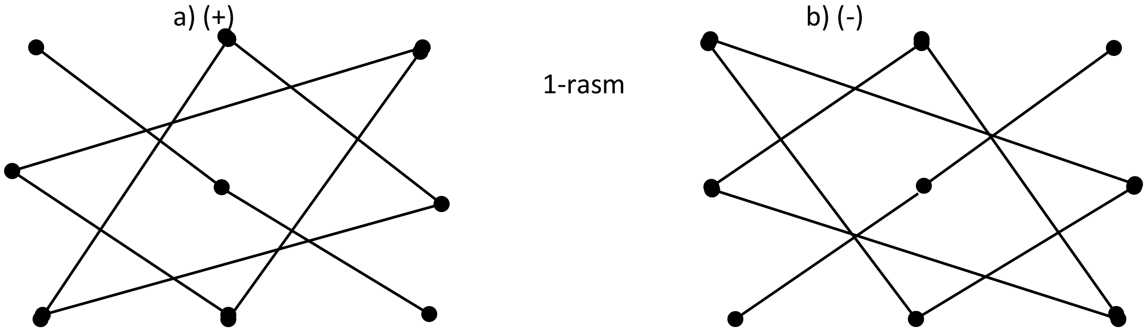
$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{12}a_{23}a_{31} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

(2)

LITERATURE ANALYSIS AND METHODOLOGY

Determinants (1) and (2) are also called 2nd-order and 3rd-order determinants, respectively. (2) the following diagram, called the "method of triangles", can be used to calculate the determinant:



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In each diagram, the connected elements are multiplied together, and then the results are added,

a) sum in the diagram with a "+" sign,

b) the sum in the diagram is taken with a "-" sign, and both results are added together.

Properties:

1. If all path elements of the determinant are replaced by column elements or vice versa, its value does not change:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

2. If we replace two adjacent row (column) elements of the determinant accordingly, the value of the determinant changes to the opposite sign:

$$\begin{vmatrix} a_{12} & a_{11} \\ a_{22} & a_{21} \end{vmatrix} = a_{12}a_{21} - a_{11}a_{22} = -\left(a_{11}a_{22} - a_{12}a_{21}\right) = -\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

3. If any row (column) elements of the determinant have a common multiplier, then this multiplier can be removed from the determinant:

$$\begin{vmatrix} a_{11} & \lambda a_{12} \\ a_{21} & \lambda a_{22} \end{vmatrix} = \lambda a_{11}a_{22} - \lambda a_{12}a_{21} = \lambda\left(a_{11}a_{22} - a_{12}a_{21}\right) = \lambda \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

4. If some row (column) elements of the determinant are proportional to other row (column) elements, then the value of the determinant is equal to zero:

$$\begin{vmatrix} a_{11} & \lambda a_{11} \\ a_{21} & \lambda a_{21} \end{vmatrix} = \lambda a_{11} a_{21} - \lambda a_{11} a_{21} = \lambda \left(a_{11} a_{21} - a_{11} a_{21} \right) = \lambda \begin{vmatrix} a_{11} & a_{11} \\ a_{21} & a_{21} \end{vmatrix} = 0$$

In particular, if $\lambda = 0$, the value of the determinant is equal to zero.

5. If the row (column) elements of the determinant are in the form of the sum of two expressions, then the determinant can be written in the form of the sum of two determinants:

$$\begin{vmatrix} a_{11} & a_{12}^{1} + a_{12}^{11} \\ a_{21} & a_{22}^{1} + a_{22}^{11} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12}^{1} \\ a_{21} & a_{22}^{1} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12}^{11} \\ a_{21} & a_{22}^{11} \end{vmatrix}$$

6. If we multiply the elements of a row (column) of the determinant by a number $\lambda \neq 0$ and add them to other row (column) elements, the value of the determinant will not change:

$$\begin{vmatrix} a_{11} & a_{12} + \lambda a_{11} \\ a_{21} & a_{22} + \lambda a_{21} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & \lambda a_{11} \\ a_{21} & \lambda a_{21} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

The above-mentioned properties are valid even when the determinant is of the third or higher order.

DISCUSSION AND RESULTS

We use the third-order determinant △ to introduce the following properties,

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

The second-order determinant formed by deleting the *i*-row and *j*-column of the given third-order determinant is called the minor of the element a_{ij} and is denoted as M_{ij}.

For example, a_{11} is the minor of the element

$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$. Likewise, a_{12} is the minor of the element $M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ is equal to and so on.

The following expression $A_{ij} = (-1)^{i+j} M_{ij}$ is called the algebraic complement of element a_{ij}.

Algebraic complement of element a_{11} $A_{11}=\begin{vmatrix} a_{22} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$, - and the algebraic complement of a_{12} element $A_{12}=-\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ is equal to and so on.

If we add the elements of a row (column) of the determinant by multiplying their algebraic complements, then the sum is equal to the value of the determinant. For real,

$$\Delta=a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13} \qquad \Delta=a_{11}A_{11}+a_{22}A_{22}+a_{13}A_{13}$$
$$\Delta=a_{21}A_{21}+a_{22}A_{22}+a_{23}A_{23} \qquad \Delta=a_{21}A_{21}+a_{22}A_{22}+a_{23}A_{23}$$
$$\Delta=a_{31}A_{31}+a_{32}A_{32}+a_{33}A_{33} \qquad \Delta=a_{31}A_{31}+a_{23}A_{23}+a_{33}A_{33}$$

It is not difficult to prove that the equations are correct.

$$\Delta=a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}=a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}-a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}+a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}=$$
$$=a_{11}(a_{22}a_{33}-a_{23}a_{32})-a_{12}(a_{21}a_{33}-a_{23}a_{31})+a_{13}(a_{21}a_{32}-a_{22}a_{31})=$$
$$=a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}-a_{13}a_{22}a_{31}-a_{12}a_{21}a_{33}-a_{11}a_{23}a_{32}$$

If we multiply the elements of a row (column) of the determinant by multiplying the algebraic complements of the elements of the other row (column), then the sum is equal to zero. For example,

$$a_{11}A_{21}+a_{12}A_{22}+a_{13}A_{23}=0 \qquad a_{11}A_{12}+a_{21}A_{22}+a_{31}A_{32}=0$$
$$a_{11}A_{31}+a_{12}A_{32}+a_{13}A_{33}=0 \qquad a_{11}A_{13}+a_{21}A_{23}+a_{31}A_{33}=0$$

and so on. For real,

$$a_{11}A_{31}+a_{12}A_{32}+a_{13}A_{33}=a_{11}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}-a_{12}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}+a_{13}\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}=$$
$$=a_{11}a_{12}a_{23}-a_{11}a_{13}a_{22}-a_{11}a_{12}a_{23}+a_{12}a_{13}a_{21}+a_{11}a_{13}a_{22}-a_{12}a_{13}a_{21}=0$$

The above-mentioned properties are also valid for n-order determinants introduced below.

Matching any π to the set {1, 2, …, n} of the first n natural numbers is called an n-ordered permutation. Any n-order π permutation can be written as:

$\pi=\begin{pmatrix} i_1 & i_2 & \dots i_n \\ \alpha_{i_1} & \alpha_{i_2} & \dots\alpha_{i_n} \end{pmatrix}$ in particular, $\pi=\begin{pmatrix} 1 & 2 & \dots n \\ \alpha_1 & \alpha_2 & \dots\alpha_n \end{pmatrix}$ **is called canonical placement.**

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If $i<j$ and if $a_i>a_j$, *We say that the pair (i, j) in the permutation π forms an inversion*. If the number of all inverse pairs $S(\pi)$ is even, π placement is even, if $S(\pi)$ is odd, π placement is called odd.

An example. The following $\pi=\begin{pmatrix}1 & 3 & 5 & 2 & 4\\ 2 & 3 & 5 & 4 & 1\end{pmatrix}$ determine whether the placement is even or odd.

Solving. We record the given placement in canonical form:

$\pi=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 2 & 4 & 3 & 1 & 5\end{pmatrix}$ and count the number of inversions. Since the inverse pairs are (1, 4), (2, 3), (2, 4), (3, 4), $S(\pi)=4$, therefore, π- is an even arrangement.

Description. The following

$A=\begin{pmatrix}a_{11} & a_{12}\dots a_{1n}\\ a_{21} & a_{22}\dots a_{2n}\\ \dots & \dots \quad \dots\\ a_{n1} & a_{n2}\dots a_{nn}\end{pmatrix}$ The n-order determinant of a square matrix is called the following number:

$$\det A=\begin{vmatrix}a_{11} & a_{12}\dots a_{1n}\\ a_{21} & a_{22}\dots a_{2n}\\ \dots & \dots \quad \dots\\ a_{n1} & a_{n2}\dots a_{nn}\end{vmatrix}=\sum_{\pi}(-1)^{S(\pi)}a_{1,\pi(1)}\dots a_{n,\pi(n)}$$

here the summation is performed over all n-order placements.

To understand this definition, consider the case where n = 3. All 3-order placements are:

$$\pi_1=\begin{pmatrix}1\,2\,3\\ 1\,2\,3\end{pmatrix},\quad \pi_2=\begin{pmatrix}1\,2\,3\\ 3\,1\,2\end{pmatrix},\quad \pi_3=\begin{pmatrix}1\,2\,3\\ 2\,3\,1\end{pmatrix},\quad \pi_4=\begin{pmatrix}1\,2\,3\\ 3\,2\,1\end{pmatrix},\quad \pi_5=\begin{pmatrix}1\,2\,3\\ 2\,1\,3\end{pmatrix},\quad \pi_6=\begin{pmatrix}1\,2\,3\\ 1\,3\,2\end{pmatrix}$$

Calculating the number of inversions for each placement: $S(\pi_1)=0$, $S(\pi_2)=2$, $S(\pi_3)=2$, $S(\pi_4)=3$, $S(\pi_5)=1$, $S(\pi_6)=1$ we make sure it is. Then by definition:

$$\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}=a_{11}a_{22}a_{33}+a_{21}a_{32}a_{13}+a_{12}a_{23}a_{31}-a_{13}a_{22}a_{31}-a_{12}a_{21}a_{33}-a_{11}a_{23}a_{32}$$

as a result, we created the previously presented formula for the 3rd-order determinant.

CONCLUSION

Similarly to the above, it is possible to introduce an algebraic complement for the determinant of order n. Then all properties of 2nd and 3rd order determinants are valid for n-order determinants. In particular,

$$\det A=\sum_{k=1}^{n} a_{ik}A_{ik} \quad (i=1,\dots,n) \tag{3}$$

$$\det A=\sum_{i=1}^{n} a_{ik}A_{ik} \quad (k=1,\dots,n) \tag{4}$$

where A_{ik} algebraic complements are determinants of order n – 1, therefore, formulas (3), (4) are also called the method of reducing the order of calculating the n-order determinant or spreading it by row and column elements.

An example. Calculate: $\begin{vmatrix} 2 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ 3 & -1 & 2 & 3 \\ 3 & 1 & 6 & 1 \end{vmatrix}$

Solving. For example, we first add the elements of column 3 to column 2 and multiply by (-2) to column 1:

$$\begin{vmatrix} 0 & 0 & 1 & 0 \\ -4 & 3 & 2 & -1 \\ -1 & 1 & 2 & 3 \\ -9 & 7 & 6 & 1 \end{vmatrix} = 1\cdot\begin{vmatrix} -4 & 3 & -1 \\ -1 & 1 & 3 \\ -9 & 7 & 1 \end{vmatrix}$$

If we multiply the 3rd column by (-4) and 3 and add it to the 1st and 2nd columns, respectively:

$$\begin{vmatrix} 0 & 0 & -1 \\ -13 & 10 & 3 \\ -13 & 10 & 1 \end{vmatrix} = 1\cdot\begin{vmatrix} -13 & 10 \\ -13 & 10 \end{vmatrix} = 0$$

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