

## O'QUVCHILARGA TRIGONOMETRIK TENGLAMALARNI YECHISHNING NOSTANDART USULLARINI O'RGATISH

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### **Annotatsiya**

Ushbu maqolada ba'zi trigonometric tenglamalarni yechishning sun'iy shakl almashtirishlardan, matematik tahlil elementlaridan, vektorlarning skalyar ko'paytmasidan foydalanish usullari keltirilgan. Ba'zi trigonometrik ifodalar qatnashgan transendent tenglamalarni yechishning nostandart usullari keltirilgan.

### **Kalit so'zlar**

trigonometriya, tenglama, matematik tahlil, sun'iy shakl almashtirish, vektorlar, skalyar ko'paytma, transendent tenglama

Ba'zi trigonometrik tenglamalarni yechishda nostandart usullardan foydalanish ancha qulaylik tug'diradi. Ular quyidagilar:

I. Sun'iy shakl almashtirishlardan foydalanib trigonometrik tenglamalarni yechish

II. Trigonometrik tenglamalarni yechishda matematik tahlil elementlaridan foydalanish

III. Trigonometrik tenglamalarni yechishda vektorlarning skalyar ko'paytmasidan foydalanish

IV. Ba'zi trigonometrik ifodalar qatnashgan transendent tenglamalarni yechish.

I. Sun'iy shakl almashtirish talab qiladigan trigonometrik tenglamalarni yechishda quyidagi usullardan foydalaniladi.

1. Tenglamaning har ikki tomonini bir xil trigonometrik funksiyaga ko'paytirish.

2. Tenglamaning har 2 tomoniga bir xil son yoki bir xil trigonometrik funksiyani qo'shish.

3. Tenglamaning biror bir qismini ayniy shakl almashtirish. (Bir xil ifodani qo'shish yoki ayirish).

$$4. \frac{a}{b} = \frac{c}{d} \text{ proporsiyadan foydalanish.}$$

Masalan.

1-misol.  $tgx \cdot tg2x = tg3x \cdot tg4x$  tenglamani yeching.

Yechish: Tenglamaning aniqlanish sohasi

$$x \neq \frac{\pi}{2} + \pi n, \quad x \neq \frac{\pi}{4} + \frac{\pi n}{2}; \quad x \neq \frac{\pi}{6} + \frac{\pi n}{3}; \quad x \neq \frac{\pi}{8} + \frac{\pi n}{4}; \quad n \in Z$$

Tenglamaning har 2 tomoniga 1 ni qo'shamiz.  $tgx \cdot tg2x + 1 = tg3x \cdot tg4x + 1.$

$$\frac{\cos x}{\cos x \cos 2x} = \frac{\cos x}{\cos 3x \cos 4x}. \text{ tenglamaning har 2 tomonini } \cos x \neq 0 \text{ ga bo'lamiz.}$$

$\cos x + \cos 3x = \cos 7x + \cos x$  ushbu tenglamani yechib quyidagi yechimni topamiz.

Javob:  $x = \frac{\pi}{5}t; \quad t \in Z.$

II. Trigonometrik tenglamalarni yechishda matematik tahlil elementlaridan foydalanish.

1. Funksiyaning aniqlanish sohasidan foydalanish
2. Funksiyaning chegaralanganlik xossasidan foydalanish
3. Sinus va kosinus funksiyalar xossalaridan foydalanish.
4. Sonli tengsizliklardan fodalanih.

2-misol.  $\left(\frac{1}{\sin^8 x} + \frac{1}{\cos^2 2x}\right)(\sin^8 x + \cos^2 2x) = 4 \cos^2 \sqrt{\frac{\pi^2}{4} - x^2}$  tenglamani yeching.

Yechish: Ixtiyoriy musbat  $a$  va  $b$  sonlari uchun  $\left(\frac{1}{a} + \frac{1}{b}\right)(a+b) \geq 4$  tengsizlik

o'rinli ekani ma'lum. Bundan 
$$\begin{cases} \left(\frac{1}{\sin^8 x} + \frac{1}{\cos^2 2x}\right)(\sin^8 x + \cos^2 2x) = 4 \\ \cos^2 \sqrt{\frac{\pi^2}{4} - x^2} = 1 \end{cases} \text{ sistema}$$

ma'lum.  $x_1 = \frac{\pi}{2}$  va  $x_2 = -\frac{\pi}{2}$  lar berilgan tenglamaning yechimlari bo'ladi. J:  $x_1 = \frac{\pi}{2},$

$$x_2 = -\frac{\pi}{2}$$

III. Trigonometrik tenglamalarni yechishda vektorlarning skalyar ko'paytmasidan foydalanish

Ma'lumki, 2 ta vektorning skalyar ko'paytmasi ularning uzunliklari va ular orasidagi burchak kosinusi ko'paytmasiga teng.  $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos \alpha$ .

$$|\cos \alpha| \leq 1, \text{ bo'lgani uchun } |\vec{a} \cdot \vec{b}| \leq |\vec{a}| \cdot |\vec{b}|.$$

Agar vektorlar koordinatalari bilan berilgan bo'lsa, ya'ni  $\vec{a}\{a_1, a_2\}$  va  $\vec{b}\{b_1, b_2\}$  bo'lsa,  $a_1, a_2 + b_1 b_2 \leq \sqrt{a_1^2 + a_2^2} \cdot \sqrt{b_1^2 + b_2^2}$  munosabat o'rinli.

3-misol.  $\sin x \sqrt{\sin x} + \cos x \sqrt{\cos x} = \sqrt{\sin x + \cos x}$  tenglamani yeching.

Yechish:  $\vec{a}\{\sin x; \cos x\}$  va  $\vec{b}\{\sqrt{\sin x}; \sqrt{\cos x}\}$  bo'lsin. U holda

$$\sin x \sqrt{\sin x} + \cos x \sqrt{\cos x} \leq \sqrt{\sin x + \cos x}$$

Tenglik belgisi  $\frac{\sin x}{\sqrt{\sin x}} = \frac{\cos x}{\sqrt{\cos x}}$  yoki  $\sqrt{\sin x} = \sqrt{\cos x}$  bo'lganda o'rinli.

$$\text{Bundan } x = \frac{\pi}{4} + 2\pi n; \quad n \in \mathbb{Z}.$$

IV. Ba'zi trigonometrik ifodalar qatnashgan transcendent tenglamalarni yechish.

4-misol.  $\sqrt{3} \sin a^x + \cos a^x = b$  tenglamani yeching.

$$\text{Yechish: } \sin(a^x + \frac{\pi}{6}) = \frac{b}{2}$$

Agar  $|b| > 2$  bo'lsa,  $\sin(a^x + \frac{\pi}{6}) = \frac{b}{2}$  tenglama yechimga ega emas, agar  $|b| \leq 2$  bo'lsa,  $a^x = (-1)^n \arcsin \frac{b}{2} - \frac{\pi}{6} + \pi n$  tenglama quyidagi 2 ta tenglamalar birlashmasiga teng kuchli:

$$a^x = \arcsin \frac{b}{2} - \frac{\pi}{6} + 2\pi k$$

$$a^x = \frac{5\pi}{6} - \arcsin \frac{b}{2} + 2\pi k$$

$a^x > 0$  tengsizlikdan ushbu tenglamalar ildizga ega bo'ladigan shart aniqlanadi.

1.  $a^x = \arcsin \frac{b}{2} - \frac{\pi}{6} + 2\pi k$  tenglama uchun  $\arcsin \frac{b}{2} - \frac{\pi}{6} + 2\pi k > 0$  bundan

$$k = \begin{cases} 0, 1, 2, \dots, m, \dots \text{ agar } 1 < b \leq 2 \\ 1, 2, \dots, m, \dots \text{ agar } -2 \leq b \leq 1 \end{cases}$$

2.  $a^x = \frac{5\pi}{6} - \arcsin \frac{b}{2} + 2\pi k$  tenglama uchun  $\frac{5\pi}{6} - \arcsin \frac{b}{2} + 2\pi k > 0$  bundan

$$k = 0, 1, 2, \dots, m$$

3. Agar  $|b| \leq 2$  bo'lsa, tenglama 2 ta  $x = \log_a(\arcsin \frac{b}{2} - \frac{\pi}{6} + 2\pi k)$  va

$$x = \log_a(\frac{5\pi}{6} - \arcsin \frac{b}{2} + 2\pi k)$$
 yechimlarga ega.

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